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Alternative approach to $B^- \to \eta' K^-$ branching ratio calculation

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Abstract. Since the calculation of BR$(B^- \to \eta' K^-)$ in the framework of the QCD improved factorization method developed by Beneke et al. leads to numerical values much below the experimental data, we include two different contributions, in an alternative way. First, we find that the spectator hard-scattering mechanism increases the BR value with almost 50%, but the predictions depend on the combined singularities in the amplitude convolution. Secondly, by adding SUSY contributions to the Wilson coefficients, we come to a BR depending on three parameters, whose values are constrained by the experimental data.

1 Introduction

As first evidence of a strong penguin, the $B^- \to \eta' K^-$ decay has become of a real interest after CLEO announced its large numerical value to be BR$(B^- \to \eta' K^-) = (6.5^{+1.5}_{-1.4} \pm 0.9) \times 10^{-5}$ [1], which could not be explained by the existent theoretical models. As improved measurements followed, providing even larger values, $(80^{+10}_{-9} \pm 7) \times 10^{-6}$ (CLEO [2]), $(76.9 \pm 3.5 \pm 4.4) \times 10^{-6}$ (BaBar [3]) and $(79^{+12}_{-11} \pm 9) \times 10^{-6}$ (Belle [4]), the inclusion of new contributions for accommodating these data has quickly become a real theoretical challenge. In this respect, perturbative QCD mechanisms [5], with different $\eta' g^* g^*$ vertex functions [5, 6], have been considered as the main candidates for significantly increasing the BR$(B^- \to \eta' K^-)$ value. On the other hand, while searching for physics beyond the standard model (SM), supersymmetry has been employed in processes like $B \to J/\psi K^*$ [7], $B \to \phi K$ [8], $B \to \pi K$ [9, 10], $B \to X_s \gamma$ [11], and deviations from the SM predictions for the values of branching ratios and CP asymmetries have been the main targets.

The present paper is organized as follows: in Sect. 2, we compute the BR$(B^- \to \eta' K^-)$ in the improved factorization approach developed by Beneke et al. [12]. Since we get a BR much below the experimental values, we incorporate two alternative contributions. The first one, presented in Sect. 3, comes from the so-called spectator hard-scattering mechanism. Following a similar approach as in [13], we give a detailed calculation of the gluonic transition form factor which plays an important role in the evaluation of this contribution. Although it has been concluded that this mechanism could provide large BR values [13], we show that the presence of combined singularities in the amplitude convolution is a source of large uncertainties. In Sect. 4, we employ a supersymmetric approach and include exchanges of gluino and squark with left–right squark mixing. Working in the mass insertion approximation [14], the values of the Wilson coefficients c_{8g} and $c_{7\gamma}$ can be significantly increased, by adding the SUSY contributions, and this has a strong numerical impact in the branching ratio estimation. Finally, one may use the experimental data to impose constraints on the flavor changing SUSY parameter δ^{bs}_{LR}.

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2 Improved QCD factorisation

The relevant decay amplitude for $B^- \to \eta' K^-$, in the improved QCD factorization approach [12], is given by [5, 15]

$$
\begin{aligned}
&A(B^- \to \eta' K^-) \\
&= -\mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}}(m_B^2 - m_{\eta'}^2)F_0^{B\to\eta'}(m_K^2) f_K \left[V_{ub}V_{us}^* a_1(X)\right. \\
&\quad \left. + V_{pb}V_{ps}^* \left(a_4^p(X) + a_{10}^p(X) + r_\chi^K\left(a_6^p(X) + a_8^p(X)\right)\right)\right] \\
&\quad -\mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}}(m_B^2 - m_K^2)F_0^{B\to K}(m_{\eta'}^2) f_{\eta'}^u \left[V_{ub}V_{us}^* a_2(Y)\right. \\
&\quad + V_{pb}V_{ps}^* \left[(a_3(Y) - a_5(Y))(2+\sigma)\right. \\
&\quad + \left[a_4^p(Y) - \frac{1}{2}a_{10}^p(Y) + r'_\chi\left(a_6^p(Y) - \frac{1}{2}a_8^p(Y)\right)\right]\sigma \\
&\quad \left.\left. + \frac{1}{2}(a_9(Y) - a_7(Y))(1-\sigma)\right]\right], \qquad (1)
\end{aligned}
$$

where $X = \eta' K$ and $Y = K\eta'$, p is summed over u and c, $r'_\chi = 2m_{\eta'}^2/(m_b - m_s)(2m_s)$, $r_\chi^K = 2m_K^2/m_b(m_u + m_s)$, $\sigma = f_{\eta'}^s/f_{\eta'}^u$, and [12]

$$
a_1(M_1M_2) = c_1 + \frac{c_2}{N_c}\left[1 + \frac{C_F\alpha_{\mathrm{s}}}{4\pi}(V_{M_2} + H)\right],
$$

$$a_2(M_1M_2) = c_2 + \frac{c_1}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2} + H)\right],$$

$$a_3(M_1M_2) = c_3 + \frac{c_4}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2} + H)\right],$$

$$a_4^p(M_1M_2) = c_4 + \frac{c_3}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2} + H)\right] + \frac{C_F\alpha_s}{4\pi N_c}P^p_{M_2,2},$$

$$a_5(M_1M_2) = c_5 + \frac{c_6}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(-12 - V_{M_2} - H)\right],$$

$$a_6^p(M_1M_2) = c_6 + \frac{c_5}{N_c}\left(1 - 6\frac{C_F\alpha_s}{4\pi}\right) + \frac{C_F\alpha_s}{4\pi N_c}P^p_{M_2,3},$$

$$a_7(M_1M_2) = c_7 + \frac{c_8}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(-12 - V_{M_2} - H)\right],$$

$$a_8^p(M_1M_2) = c_8 + \frac{c_7}{N_c}\left(1 - 6\frac{C_F\alpha_s}{4\pi}\right) + \frac{\alpha}{9\pi N_c}P^{p,\mathrm{EW}}_{M_2,3},$$

$$a_9(M_1M_2) = c_9 + \frac{c_{10}}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2} + H)\right],$$

$$a_{10}^p(M_1M_2) = c_{10} + \frac{c_9}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2} + H)\right] + \frac{\alpha}{9\pi N_c}P^{p,\mathrm{EW}}_{M_2,2}, \quad (2)$$

where $C_F = (N_c^2 - 1)/2N_c$ and $N_c = 3$ is the number of colors. The vertex, the hard gluon exchange with the spectator and the penguin contributions, at $\mu = m_b$, are

$$V_M = -18 + \int_0^1 \mathrm{d}x g(x)\phi_M(x),$$

$$P^p_{M,2} = c_1\left[\frac{2}{3} + G_M(s_p)\right] + c_3\left[\frac{4}{3} + G_M(0) + G_M(1)\right] + (c_4 + c_6)\left[(n_f - 2)G_M(0) + G_M(s_c) + G_M(1)\right] - 2c_{8g}^{\mathrm{eff}}\int_0^1 \frac{\mathrm{d}x}{1-x}\phi_M(x),$$

$$P^{p,\mathrm{EW}}_{M,2} = (c_1 + N_c c_2)\left[\frac{2}{3} + G_M(s_p)\right] - 3c_{7\gamma}^{\mathrm{eff}}\int_0^1 \frac{\mathrm{d}x}{1-x}\phi_M(x),$$

$$P^p_{M,3} = c_1\left[\frac{2}{3} + \hat{G}_M(s_p)\right] + c_3\left[\frac{4}{3} + \hat{G}_M(0) + \hat{G}_M(1)\right] + (c_4 + c_6)\left[(n_f - 2)\hat{G}_M(0) + \hat{G}_M(s_c) + \hat{G}_M(1)\right] - 2c_{8g}^{\mathrm{eff}},$$

$$P^{p,\mathrm{EW}}_{M,3} = (c_1 + N_c c_2)\left[\frac{2}{3} + \hat{G}_M(s_p)\right] - 3c_{7\gamma}^{\mathrm{eff}},$$

$$H = \frac{4\pi^2}{N_c}\frac{f_B f_{M_1}}{m_B^2 F_0^{B\to M_1}(0)} \times \int_0^1 \frac{\mathrm{d}\xi}{\xi}\phi_B(\xi)\int_0^1 \frac{\mathrm{d}x}{\bar{x}}\phi_{M_2}(x) \times \int_0^1 \frac{\mathrm{d}y}{\bar{y}}\left[\phi_{M_1}(y) + \frac{2\mu_{M_1}}{m_b}\frac{\bar{x}}{x}\phi^p_{M_1}(y)\right], \quad (3)$$

where $\bar{x} = 1 - x$, $\bar{y} = 1 - y$ and the parameter $2\mu_M/m_b$ coincides with r_χ. The functions $g(x)$, $G_M(x)$ and $\hat{G}_M(x)$ are given by

$$g(x) = 3\left(\frac{1-2x}{1-x}\ln x - \mathrm{i}\pi\right) + \left[2\mathrm{Li}_2(x) - \ln^2 x + \frac{2\ln x}{1-x} - (3 + 2\mathrm{i}\pi)\ln x - (x \to \bar{x})\right],$$

$$G(s,x) = 4\int_0^1 \mathrm{d}u\, u\bar{u}\ln[s - u\bar{u}x] = -\frac{10}{9} + \frac{2}{3}\ln s - \frac{8s}{3x} + \frac{4}{3}\left(1 + \frac{2s}{x}\right)\sqrt{\frac{4s}{x} - 1}\arctan\frac{1}{\sqrt{\frac{4s}{x} - 1}},$$

$$G_M(s) = \int_0^1 \mathrm{d}x\, G(s - \mathrm{i}\epsilon, \bar{x})\phi_M(x),$$

$$\hat{G}_M(s) = \int_0^1 \mathrm{d}x\, G(s - \mathrm{i}\epsilon, \bar{x})\phi^p_M(x), \quad (4)$$

where $s_i = m_i^2/m_b^2$ are the mass ratios for the quarks involved in the penguin diagrams, namely $s_u = s_d = s_s = 0$ and $s_c = (1.3/4.2)^2$.

As it can be noticed, except for the hard contribution where the wave functions for both M_1 and M_2 are involved, the coefficients a_i are different for the X and Y final states, since they depend on the twist-2 and twist-3 wave functions of the M_2 meson. Thus, the twist-2 distribution amplitude $\phi_K(x)$ has the following expansion in Gegenbauer polynomials: [12,16]

$$\phi_K(x) = 6x(1-x) \times [1 + \alpha_1^K C_1^{(3/2)}(2x-1) + \alpha_2^K C_2^{3/2}(2x-1) + \ldots], \quad (5)$$

with $C_1^{3/2}(u) = 3u$, $C_2^{3/2}(u) = (3/2)(5u^2 - 1)$, $\alpha_1^K = 0.3 \pm 0.3$, and $\alpha_2^K = 0.1 \pm 0.3$. The corresponding twist-3 amplitude, ϕ_K^p, is 1.

The physical states η and η' are mixtures of the SU(3)-singlet and octet components η_0 and η_8 and therefore the corresponding decay constants, in the two-angle mixing formalism, are given by

$$f^u_{\eta'} = \frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0,$$

$$f^s_{\eta'} = -2\frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0\,, \tag{6}$$

with $\theta_8 = -22.2°$, $\theta_0 = -9.1°$, $f_8 = 168\,\mathrm{MeV}$, and $f_0 = 157\,\mathrm{MeV}$ [17]. These lead to $f^u_{\eta'} = 63.5\,\mathrm{MeV}$, $f^s_{\eta'} = 141\,\mathrm{MeV}$ and to the relevant form factor for the $B \to \eta'$ transition

$$F_0^{B\to\eta'} = F_0^\pi\left(\frac{\sin\theta_8}{\sqrt{6}} + \frac{\cos\theta_0}{\sqrt{3}}\right) = 0.137\,. \tag{7}$$

Even though the η' flavor singlet meson has a gluonic content which could bring about a contribution to the wave function, this is supposed to be small [18] and therefore we employ, in the calculation of $V_{\eta'}$, $P^p_{\eta',2}$ and $P^{p,\mathrm{EW}}_{\eta',2}$ in $a_i(Y)$, only the leading twist-2 distribution amplitude

$$\phi_{\eta'} = 6x\bar{x}\,. \tag{8}$$

Also, since the twist-3 quark–antiquark distribution amplitude does not contribute, due to the chirality conservation, the penguin parts in $a^p_6(Y)$ and $a^p_8(Y)$ are missing. As for the B meson wave function, we shall work with a strongly peaked one, around $z_0 = \lambda_B/m_B \approx 0.066 \pm 0.029$, for $\lambda_B = 0.35 \pm 0.15\,\mathrm{GeV}$.

Putting everything together, we get, within the SM improved factorization approach [12], the numerical value $\mathrm{BR_{SM}}(B \to \eta' K) = 3.65 \times 10^{-5}$, which although it is in accordance with other theoretical estimations [5, 15, 17], yet it lies below the experimental data [1–4]. Hence, in spite of the "conservative" prediction that the conventional mechanism should be the dominant one, it has become clear that new contributions are needed in order to account for the existent data.

3 Spectator hard-scattering mechanism

It has been considered that the spectator hard-scattering mechanism (SHSM), depicted in Fig. 1, is a reliable framework for this process, which significantly increases the value of $\mathrm{BR}(B \to \eta' K)$ [5, 13]. Following this idea, let us write down the corresponding di-gluon exchange amplitude for the b quark decaying into an s quark and a hard gluon:

$$A_{hs} = -\mathrm{i}C_F g_s^3 \frac{f_B}{2\sqrt{6}}\frac{f_K}{2\sqrt{6}}\int \mathrm{d}z\,\mathrm{d}y\,\phi_B(z)\phi_K(y)$$

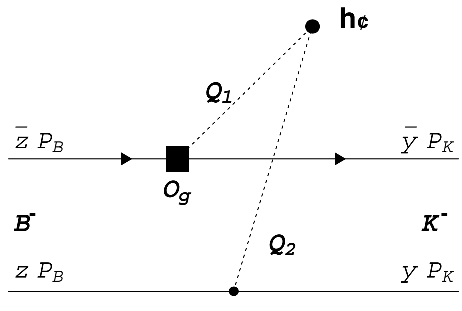

Fig. 1. Feynman diagrams of the hard-scattering mechanism for $B^- \to \eta' K^-$. The gluons are represented by the dashed lines

$$\times\mathrm{Tr}\left[\gamma_5 \not{P}_k \Gamma_\mu(\not{P}_B + m_B)\gamma_5\gamma_\nu\right]$$
$$\times\frac{\varepsilon^{\mu\nu\alpha\beta}Q_{1\alpha}Q_{2\beta}}{Q_1^2 Q_2^2}F_{\eta' g^* g^*}(Q_1^2, Q_2^2, m^2_{\eta'}) \tag{9}$$

in terms of the effective $b \to sg$ vertex [19]

$$\Gamma^a_\mu = \frac{G_\mathrm{F}}{\sqrt{2}}\frac{g_s}{4\pi^2}V^*_{ps}V_{pb}\,t^a$$
$$\times\left[F^p_1\left(Q_1^2\gamma_\mu - Q_{1\mu}\not{Q}_1\right)L - F^p_2 \mathrm{i}\sigma_{\mu\nu}Q^\nu_1 m_b R\right] \tag{10}$$

and the transition form factor [6]

$$\langle g^*_a g^*_b|\eta'\rangle = -\mathrm{i}\delta_{ab}\varepsilon^{\mu\nu\alpha\beta}\varepsilon^{a*}_\mu\varepsilon^{b*}_\nu Q_{1\alpha}Q_{2\beta}F_{\eta' g^* g^*}(Q_1^2, Q_2^2, m^2_{\eta'})\,. \tag{11}$$

The quark contribution to the $\eta' g^* g^*$ vertex

$$F_{\eta' g^* g^*}(Q_1^2, Q_2^2, m^2_{\eta'}) = 4\pi\alpha_\mathrm{s}\frac{1}{2N_c}\sum_{q=u,d,s} f^q_{\eta'}\,F(y,a)\,, \tag{12}$$

with

$$F(y,a) = \int_0^1 \mathrm{d}x\,\frac{\phi_{\eta'}(x)}{\bar{x}Q_1^2 + xQ_2^2 - x\bar{x}m^2_{\eta'} + \mathrm{i}\varepsilon} + (x \leftrightarrow \bar{x})\,,$$
$$a^2 = m^2_{\eta'}/m^2_B\,, \tag{13}$$

will play an important role in the evaluation of the amplitude A_{hs}. Performing the calculations in (9), we come to the following expression of the hard-scattering amplitude:

$$A_{hs} = -2\,\mathrm{i}\,\frac{G_\mathrm{F}}{\sqrt{2}}V^*_{ps}V_{pb}\frac{\alpha_\mathrm{s}^2}{N_c^3}f_B f_K(2f^u_{\eta'} + f^s_{\eta'})$$
$$\times\int_0^1 \mathrm{d}z\,\phi_B(z)\int_0^1 \mathrm{d}y\phi_K(y)$$
$$\times\left[F^p_1 Q_1^2\left((P_B\cdot Q_1)(P_K\cdot Q_2) - (P_K\cdot Q_1)(P_B\cdot Q_2)\right)\right.$$
$$\left.+F^p_2 m_B m_b\left((P_K\cdot Q_2)Q_1^2 - (P_K\cdot Q_1)(Q_1\cdot Q_2)\right)\right]$$
$$\times\frac{F(y,a)}{Q_1^2 Q_2^2}\,. \tag{14}$$

With the gluon momenta

$$Q_1 = \bar{z}P_B - \bar{y}P_K\,,\quad Q_2 = zP_B - yP_K\,, \tag{15}$$

and neglecting, for the moment, both $m^2_{\eta'}$ and m^2_K, the amplitude (14) becomes

$$A_{hs} = \mathrm{i}\,\frac{G_\mathrm{F}}{\sqrt{2}}V^*_{ps}V_{pb}\frac{\alpha_\mathrm{s}^2}{2N_c^3}f_B f_K(2f^u_{\eta'} + f^s_{\eta'})\frac{1}{z_0} \tag{16}$$
$$\times\int_0^1 \phi_K(y)\left[m_B^2 F^p_1 + m_B m_b\frac{F^p_2}{y - z_0}\right]F(y,a)\,,$$

where, for the dominant contribution coming from the insertion of the $O^{u,c}_1$ and the magnetic-penguin O_{8g} operators, one has [13]

$$F^p_1 = c_1\left[\frac{2}{3} + G[s_p, (1 - z_0)(y - z_0)]\right]\,,\quad F^p_2 = -2c_{8g}\,. \tag{17}$$

As far as concerns the $F(y,a)$ function, which is an essential input in the calculations, it can be first written as

$$F(y,a) = \tag{18}$$
$$4\int_0^1 \mathrm{d}x \, \frac{6x\bar{x}\,(Q_1^2+Q_2^2-2x\bar{x}m_{\eta'}^2)}{\left[Q_1^2+Q_2^2-2x\bar{x}m_{\eta'}^2\right]^2-\left[(x-\bar{x})(Q_1^2-Q_2^2)\right]^2}$$

and we are led, after algebraic computations, to the following form:

$$F(y,a) = -\frac{12}{m_{\eta'}^2}\left[1-\frac{Q_1^2-Q_2^2}{2m_{\eta'}^2}\log\left|\frac{Q_1^2}{Q_2^2}\right| + \frac{(Q_1^2-Q_2^2)^2-m_{\eta'}^2(Q_1^2+Q_2^2)}{2m_{\eta'}^2\sqrt{p^4-4Q_1^2Q_2^2}} \times \log\left|1+2\,\frac{\sqrt{p^4-4Q_1^2Q_2^2}}{p^2-\sqrt{p^4-4Q_1^2Q_2^2}}\right|\right], \tag{19}$$

where we have introduced the notation $p^2 = Q_1^2+Q_2^2-m_{\eta'}^2$. The logarithmic nature of the $F(y,a)$ function makes it very sensitive to the values of Q_1^2, Q_2^2, $m_{\eta'}^2$. We recommend [6] for a detailed discussion of the $\eta' g^* g^*$ vertex in the case of arbitrary gluon virtualities in the time-like, $Q_1^2 > 0$, $Q_2^2 > 0$, $p^4 - 4Q_1^2Q_2^2 > 0$, and space-like, $Q_1^2 < 0$, $Q_2^2 < 0$, $p^4 - 4Q_1^2Q_2^2 < 0$, regions.

Now, using

$$Q_1^2 \approx \bar{z}\left[(y-z)m_B^2+\bar{y}m_{\eta'}^2\right], \quad Q_2^2 \approx z\left[-(y-z)m_B^2+ym_{\eta'}^2\right], \tag{20}$$

where we have neglected m_K^2, the dominant term in (19) is

$$F(y,a) \approx -\frac{12}{m_{\eta'}^2}\left[1-\frac{1}{2}\left[\frac{y-z}{a^2}+(1-y-z)\right] \times \log\left|\frac{a^2+y-z}{z(z-y)}\right| + \frac{(y-z)a^2+(y-z)^2}{2a^2\,|y-z|} \times \log\left|\frac{y(1-a^2)-z+|y-z|}{y(1-a^2)-z-|y-z|}\right|\right]. \tag{21}$$

On the other hand, by comparing the expressions in (20), we clearly have the result that we are in the limit where $|Q_1^2| \gg |Q_2^2|$. So, the function $F(y,a)$ can be computed in this approximation and it simply yields

$$F(y,a) = -\frac{12}{m_{\eta'}^2}\left[1+\left(\frac{y-z_0}{a^2}+\bar{y}\right)\log\left|1-\frac{1}{\frac{y-z_0}{a^2}+\bar{y}}\right|\right]. \tag{22}$$

As it can be seen from (20), the term $(y-z_0)/a^2+\bar{y} = Q_1^2/m_{\eta'}^2$ takes a whole range of values, from -0.87 to 26.5, as Q_1^2 goes from the space-like to the time-like regions. Consequently, a logarithmic singularity develops as $y \to z_0/(1-a^2)$, i.e. for $Q_1^2 \to m_{\eta'}^2$. Inspecting (16), we also notice the pole at $y = z_0$ in the F_2^p contribution. In addition, while $G[s_p,(1-z_0)(y-z_0)]$ is divergence free for all $s > 0$, the $G[0,(1-z_0)(y-z_0)]$ gets a logarithmic singularity at $y = z_0$. Hence, in the course of numerically evaluating the scattering contribution, one must be careful about dealing with these combined singularities in the convolution (16).

As in the case of other hard-scattering theoretical estimations [5,13], the amplitude of this contribution contains, as main uncertainty, the peaking position, z_0, in the B meson distribution function and accordingly, the branching ratio is extremely sensitive to it. For $z_0 \in [0.063,\,0.068]$ and the average value $\alpha_s(Q_1^2) = 0.28$, the total branching ratio, including besides the improved factorization approach the spectator hard-scattering mechanism with the vertex function (22), is in the range from $\mathrm{BR}(B \to \eta' K) = 6.58\times10^{-5}$, for $z_0 = 0.063$, to $\mathrm{BR}(B \to \eta' K) = 5.8\times10^{-5}$, for $z_0 = 0.068$.

Comparing these results with the experimental data [1–3], we notice that they are still below the lowest limit. An alternative way which increases the BR and avoids the uncertainties coming from the combined singularities in the convolution (16) would presumably look more reliable.

4 SUSY gluonic dipole contribution

Employing the minimal supersymmetric standard model (MSSM), one may add to the effective SM Hamiltonian (1) the following SUSY contributions:

$$H_{3-6}^{\mathrm{SUSY}} = -\mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}}(V_{ub}V_{us}^*+V_{cb}V_{cs}^*)\sum_{i=3}^{6}c_i^{\mathrm{SUSY}}O_i \tag{23}$$

and

$$H_{7-8}^{\mathrm{SUSY}} = -\mathrm{i}\frac{G_{\mathrm{F}}}{\sqrt{2}}(V_{ub}V_{us}^*+V_{cb}V_{cs}^*)\left(c_{8g}^{\mathrm{SUSY}}O_{8g}+c_{7\gamma}^{\mathrm{SUSY}}O_{7\gamma}\right), \tag{24}$$

expressed in terms of the usual standard model operators O_i and the gluon and photon operators

$$O_{8g} = \frac{g_s}{8\pi^2}m_b\bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b\,, \quad O_{7\gamma} = \frac{e}{8\pi^2}m_b\bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b\,,\,. \tag{25}$$

The Wilson coefficients are given by [10,20]

$$c_3^{\mathrm{SUSY}}(M_{\mathrm{SUSY}}) = -\frac{\alpha_s^2}{2\sqrt{2}\,G_{\mathrm{F}}(V_{ub}V_{us}^*+V_{cb}V_{cs}^*)m_{\tilde{q}}^2}\,\delta_{LL}^{bs} \times\left(-\frac{1}{9}B_1(x)-\frac{5}{9}B_2(x)-\frac{1}{18}P_1(x)-\frac{1}{2}P_2(x)\right),$$

$$c_4^{\mathrm{SUSY}}(M_{\mathrm{SUSY}}) = -\frac{\alpha_s^2}{2\sqrt{2}\,G_{\mathrm{F}}(V_{ub}V_{us}^*+V_{cb}V_{cs}^*)m_{\tilde{q}}^2}\,\delta_{LL}^{bs}$$

$$\times \left(-\frac{7}{3} B_1(x) + \frac{1}{3} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right),$$

$$c_5^{\mathrm{SUSY}}(M_{\mathrm{SUSY}}) = -\frac{\alpha_s^2}{2\sqrt{2}\, G_{\mathrm{F}}(V_{ub}V_{us}^* + V_{cb}V_{cs}^*) m_{\tilde{q}}^2}\, \delta_{LL}^{bs}$$

$$\times \left(\frac{10}{9} B_1(x) + \frac{1}{18} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right),$$

$$c_6^{\mathrm{SUSY}}(M_{\mathrm{SUSY}}) = -\frac{\alpha_s^2}{2\sqrt{2}\, G_{\mathrm{F}}(V_{ub}V_{us}^* + V_{cb}V_{cs}^*) m_{\tilde{q}}^2}\, \delta_{LL}^{bs}$$

$$\times \left(-\frac{2}{3} B_1(x) + \frac{7}{6} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right), \quad (26)$$

where the $P_1(x), P_2(x), B_1(x), B_2(x)$ functions, coming from the gluino penguins and box diagrams, can be found in [14] and

$$c_{8g}^{\mathrm{SUSY}}(M_{\mathrm{SUSY}}) = -\frac{\sqrt{2}\pi\alpha_s}{G_{\mathrm{F}}(V_{ub}V_{us}^* + V_{cb}V_{cs}^*) m_{\tilde{g}}^2}\, \delta_{LR}^{bs}\, \frac{m_{\tilde{g}}}{m_b}\, G_0(x),$$

$$c_{7\gamma}^{\mathrm{SUSY}}(M_{\mathrm{SUSY}}) = -\frac{\sqrt{2}\pi\alpha_s}{G_{\mathrm{F}}(V_{ub}V_{us}^* + V_{cb}V_{cs}^*) m_{\tilde{g}}^2}\, \delta_{LR}^{bs}\, \frac{m_{\tilde{g}}}{m_b}\, F_0(x), \quad (27)$$

where

$$G_0(x) = \frac{x}{3(1-x)^4} \times \left[22 - 20x - 2x^2 + 16x\ln(x) - x^2\ln(x) + 9\ln(x) \right],$$

$$F_0(x) = -\frac{4x}{9(1-x)^4} \times \left[1 + 4x - 5x^2 + 4x\ln(x) + 2x^2\ln(x) \right]. \quad (28)$$

In the above relations, $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$, with $m_{\tilde{g}}$ being the gluino mass and $m_{\tilde{q}}$ an average squark mass, while the factor $\delta^{bs} = \Delta^{bs}/m_{\tilde{q}}^2$, where Δ^{bs} are the off-diagonal terms in the sfermion mass matrices, comes from the expansion of the squark propagator in terms of δ, for $\Delta \ll m_{\tilde{q}}^2$. In principle, the dimensionless quantities δ^{bs}, measuring the size of the flavor changing interaction for the $\tilde{s}\tilde{b}$ mixing, are present in all the SUSY corrections to the Wilson coefficients in (1) and they are of four types, depending on the L or R helicity of the fermionic partners. A simultaneous analysis, in the full parameter space, for both the LL and LR squark mixings, is difficult to perform. However, when dealing with the SUSY contributions to the Wilson coefficients, one finds major differences between them. In this respect, by computing $\{c_i^{\mathrm{SUSY}}\}_{i=\overline{3,6}}$, for $M_{\mathrm{SUSY}} = m_{\tilde{q}} = 500\,\mathrm{GeV}$ and $x \approx 1$, we have noticed that these corrections can be neglected. Since the ratios of their values and the SM Wilson coefficients are in the range 10^{-8} to 10^{-6}, they certainly do not bring about any significant contribution to the branching ratio. The situation looks different as far as concerns the SUSY Wilson coefficients (27) which are going to play an important role in the next discussion. Indeed, by comparing the expressions (26) and (27), we notice an enhancement factor of $m_{\tilde{g}}/m_b$ in (27). When $m_{\tilde{g}}$ is of the order of a few hundred GeV, these SUSY contributions will dominate the SM Wilson coefficients, which are proportional to m_b/m_W^2, and one can anticipate a large effect on the branching ratio, even for small values of δ_{LR}.

By considering only the SUSY corrections (27), we replace, in (3), the Wilson coefficients c_{8g}^{eff} and $c_{7\gamma}^{\mathrm{eff}}$, by the total quantities

$$c_{8g}^{\mathrm{total}}[x,\delta] = c_{8g}^{\mathrm{eff}} + c_{8g}^{\mathrm{SUSY}}(m_b),$$
$$c_{7\gamma}^{\mathrm{total}}[x,\delta] = c_{7\gamma}^{\mathrm{eff}} + c_{7\gamma}^{\mathrm{SUSY}}(m_b), \quad (29)$$

where $c^{\mathrm{SUSY}}(m_b)$ have been evolved from $M_{\mathrm{SUSY}} = m_{\tilde{g}}$ down to the $\mu = m_b$ scale, using the relations [10, 19]

$$c_{8g}^{\mathrm{SUSY}}(m_b) = \eta c_{8g}^{\mathrm{SUSY}}(m_{\tilde{g}}), \quad (30)$$
$$c_{7\gamma}^{\mathrm{SUSY}}(m_b) = \eta^2 c_{7\gamma}^{\mathrm{SUSY}}(m_{\tilde{g}}) + \frac{8}{3}(\eta - \eta^2) c_{8g}^{\mathrm{SUSY}}(m_{\tilde{g}}),$$

with

$$\eta = \left(\alpha_s(m_{\tilde{g}})/\alpha_s(m_t)\right)^{2/21} \left(\alpha_s(m_t)/\alpha_s(m_b)\right)^{2/23}. \quad (31)$$

We choose for $m_{\tilde{q}}$ the value $m_{\tilde{q}} = 500\,\mathrm{GeV}$ and write $m_{\tilde{g}}$ as $m_{\tilde{g}} = \sqrt{x}\, m_{\tilde{q}}$ and $\delta_{LR}^{bs} \equiv \rho e^{i\varphi}$. As the total branching ratio can be expressed in terms of three free parameters: x, ρ, φ, one is able to plot the $\mathrm{BR}^{\mathrm{total}}$, in units of 10^{-5}, as a function of (ρ, φ), for different values of x. By inspecting the 3D plots displayed in Fig. 2, for $x = 0.3$ (the upper surface) and $x = 1$ (the lower surface), we notice that the SUSY contributions (27) to the Wilson coefficients have significantly increased the SM value, $\mathrm{BR}_{\mathrm{SM}} = 3.65 \times 10^{-5}$, represented by the horizontal plane. Using the experimental data, one is able now to determine the δ_{LR}^{bs} complex values, for each x.

Let us take, for example, $x = 1$, pointing out that the same discussion can be performed for any x value. For $\rho = 0.005$, the $\mathrm{BR}^{\mathrm{total}}$ is increasing from 5.1×10^{-5}, for $\varphi \approx \pm\pi/3$, to the maximum value $\mathrm{BR}^{\mathrm{total}} = 6.24 \times 10^{-5}$, for $\varphi = 0$. As ρ goes to bigger values, we find a better agreement with the large experimental data. For $\rho = 0.01$, the data can be accommodated for $\varphi \approx -\pi/4$, while, for $\rho = 0.02$, one has to impose $\varphi \approx -8\pi/15$.

Finally, let us compare these results with the constraints given by other measurements, such as $b \to s\gamma$ decay. Using the experimental range for $\mathrm{BR}(B \to X_s\gamma)$, the allowed region, with 95% C.L., for $c_{8g}^{\mathrm{SUSY}}/c_{8g}^{\mathrm{SM}} \equiv r\, e^{i\delta}$ is discussed, in detail, in [10]. In our approach, the pair $\{\rho = 0.01,\ \varphi = -\pi/4\}$, for which the branching ratio coincides with the average data value, leads to $c_{8g}^{\mathrm{SUSY}}/c_{8g}^{\mathrm{SM}} = -5.014 + 4.85\,i$, which means, turning to the parameters r and δ from [10], $r = 6.98$ and $\delta = 136°$. These values fit perfectly to the allowed region. As ρ goes to 0.02 and the average experimental data impose $\varphi = -8\pi/15$, the ratio $c_{8g}^{\mathrm{SUSY}}/c_{8g}^{\mathrm{SM}} = 1.23 + 13.9\,i$ leads to $r = 13.95$ and $\delta = 85°$, which are outside the boundaries of the constraint.

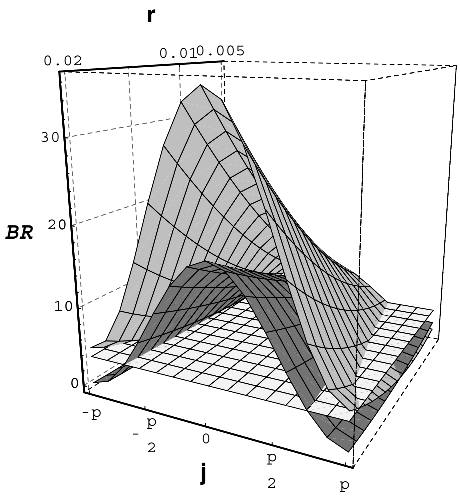

Fig. 2. Total branching ratios (SM + SUSY) for $B^- \to \eta' K^-$, in units of 10^{-5}, as functions of (ρ, φ), for $x = 0.3$ (the upper plot) and $x = 1$ (the lower plot), compared to the SM estimation represented by the horizontal plane

5 Concluding remarks

At first, we have analyzed the $B^- \to \eta' K$ decay and we have computed its branching ratio using the improved factorization method developed by Beneke et al. [12]. Since the obtained result, $\mathrm{BR_{SM}} = 3.65 \times 10^{-5}$, is much below the experimental data, [1–4], one may consider this as a clear sign for adding new contributions [22].

In this respect, the so-called spectator hard-scattering mechanism, which is depicted in Fig. 1, has allowed us to compute the amplitude in terms of the effective $b \to sg$ vertex and the transition form factor (11) which contains the quark contribution to the $\eta' g^* g^*$, (21), as an essential input. The total BR has, as a main uncertainty, the peaking position in the B meson wave function, $z_0 = \lambda_B / m_B$, with $\lambda_B = 0.35 \pm 0.15$ GeV. Even though the results are closer to the experimental data, we point out the combined singularities in the amplitude convolution (16) which must be treated carefully.

Secondly, we extend the SM to the MSSM. As the gluonic dipole interactions can be significantly enhanced compared to the SM, by the factor $m_{\tilde{g}}/m_b$, we add SUSY contributions to the Wilson coefficients c_{8g}^{eff} and $c_{7\gamma}^{\mathrm{eff}}$. Consequently, the total BR is expressed in terms of the parameters $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$, and $\delta_{LR}^{bs} = \rho \mathrm{e}^{\mathrm{i}\varphi}$ whose contribution turns out to be important, even for very small values of ρ. Finally, by inspecting the 3D-graphics (see Fig. 2), representing the $\mathrm{BR}^{\mathrm{total}}$ for $x = 0.3$ (the upper surface) and $x = 1$ (the lower surface), one is able to find numerical values for ρ and φ that can account for the experimental data and agree with the constraint coming from the $b \to s\gamma$ decay.

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